*International Journal of Theoretical Physics, Vol. 36, No. 8, 1997*

# $Z'$ Boson in the $SO(10)$ Model

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*Received September 24, 1994*

The $SO(10)$ model is a candidate for the unification of electromagnetic, weak, and strong interactions. The range of the $Z'$ mass is 495 GeV $< m_{Z'} <$ $10^9$ GeV. The formulas for the width and asymmetry for $Z'$ decay depend only on the $Z'$ mass. We apply the method of Boudjema *et al.* to identify a theoretical origin of the $Z'$ boson in $SO(10)$ and compare with other models.

## 1. INTRODUCTION

As one of the major accomplishments in particle physics, the Standard Model (SM) (Weinberg, 1967; Salam, 1969; Glashow *et al.*, 1970) is compatible with experimental data and various low-energy neutral current processes (e.g., Langacker and Mann, 1989; Langacker, 1990a,b; Amaldi *et al.*, 1987; Kim *et al.*, 1981; Costa *et al.*, 1988; Foli and Haidt, 1988). However, many questions cannot be answered within the framework of the SM. For example, there are 21 undetermined parameters; the Higgs particle is put in the theory by hand and has not yet been found in experiment; why is the gauge structure a product of three gauge groups rather than a single group? etc.

In last few years, it was found that there are some highly precise experimental results contradicting the SM. This suggests that we should look beyond the SM. Among the several new models, the $SO(10)$ model and the supersymmetric $SU(5)$ model are basically considered to be candidates for a unified theory, which needs further study. In this paper, we use the strategy developed by Boudjema *et al.* to study the theoretical origin of the $Z'$ boson, and compare the results between the $SO(10)$ model and six other models.

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0020-7748/97/0800-1753$12.50/0 

## 2. THE MASS SCALE OF $Z'$ BOSON IN THE $SO(10)$ MODEL

The $SO(10)$ model (Rajpoot, 1980) includes one extra fermion, one extra neutral gauge boson ($Z'$ boson), and 32 extra nonneutral gauge bosons compared to the SM. The one extra fermion can be naturally interpreted as the right-handed neutrino $\nu_R$ when the neutrino mass is not zero. In order to describe the electromagnetic, weak, and strong interactions in the $SO(10)$ model, the pattern of the spontaneous symmetry breaking (SSB) should be:

$$\begin{aligned} SO(10): \quad & M_G \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & M_R \quad SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ & M_{Z'} \quad SU(3)_c \times SU(2)_L \times U(1)_Y \\ & M_Z \quad SU(3)_c \times U(1)_{\text{em}} \end{aligned} \tag{1}$$

If we adopt the experiment values for $q^2 = M_Z^2$ of $\alpha_3 = 0.1134$ and $\alpha^2 = 0.0337$, two straight lines of the running coupling constants for $SU(2)$ and $SU(3)$ based on the renormalization group equations meet at (Rosner, 1991)

$$M_G \sim 10^{17}\ \text{GeV} \tag{2}$$

as illustrated in Fig. 1 [the same as Rosner (1991), Fig. 18(b)]. This means that the mass scale of the 30 extra nonneutral gauge bosons is about $10^{17}$ GeV. They are too heavy, and no accelerator can reach that energy in the near future. So we do not study them now.

The $U(1)_R$ behaves as shown in Fig. 1, since we adopt a symmetry-breaking scheme in which $SU(3)_c$ and $U(1)_{B-L}$ merge at the $M_G$ scale. The $U(1)_R$ coupling strength becomes equal to that of $SU(2)_L$ at the $M_R$ mass scale

$$M_R \sim 10^9\ \text{GeV} \tag{3}$$

which indicates that the mass scale of the two extra nonneutral gauge bosons $w_R^{\pm}$ is about $10^9$ GeV. They are also too heavy for an accelerator to search for.

The mass scale of the $Z'$ boson will be $m_Z < m_{Z'} < 10^9$ GeV in the $SO(10)$ model. A recent experiment gave $m_{Z'} > 495$ GeV (Swarts, 1993); thus, the mass scale of $Z'$ is corrected as 495 GeV $< m_{Z'} < 10^9$ GeV. Since the low limit of $m_{Z'}$ is not too large, the $Z'$ boson has often been studied (Deshpande and Iskandar, 1979a,b, 1980, and references therein; Kang and Kim, 1976a,b, 1978, and references therein).

In recent years the $SO(10)$ model has been supported by experiment. The running gauge coupling constants have been determined, except for $g_c(m_{Z'}^2)$ for $U(1)_{B-L}$ and $g_{1R}(m_{Z'}^2)$ for $U(1)_R$:

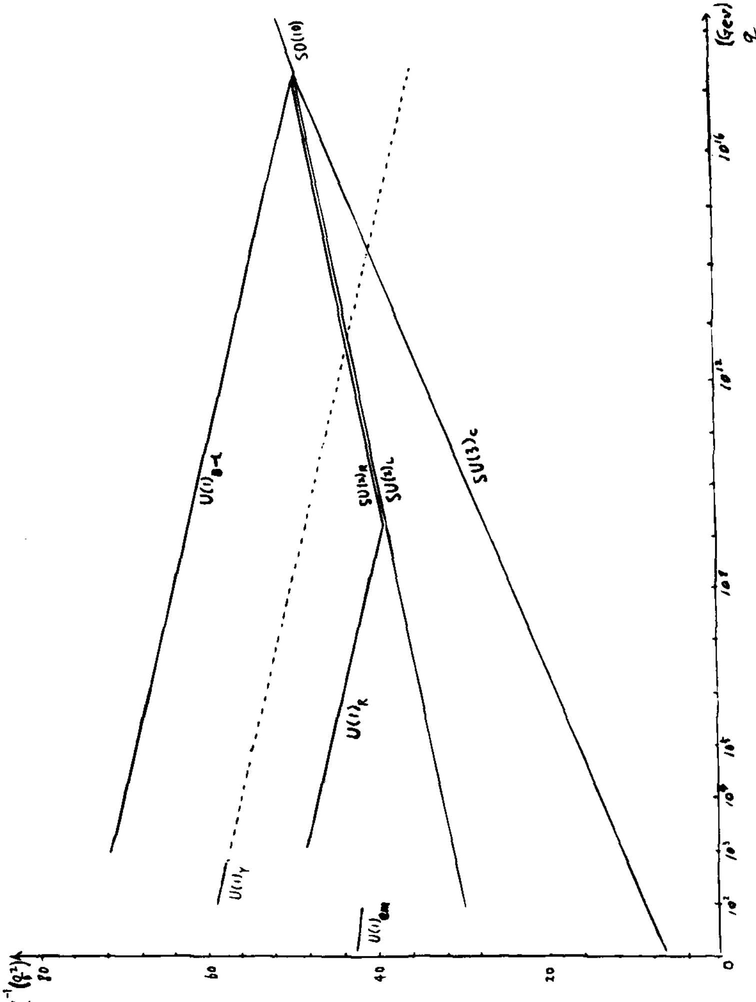


**Fig. 1.** The coupling constant $\alpha^{-1}(q^2)$ as a function of $\ln(q^2)$ in $SO(10)$ GUTs.

$$g_c^{-2}(m_{Z'}^2) = g_c^{-2}(M_G^2) - \frac{4}{(4\pi)^2} \ln\left(\frac{m_{Z'}^2}{M_G^2}\right) \tag{4}$$

$$g_{1R}^{-2}(m_{Z'}^2) = g_{1R}^{-2}(M_G^2) \frac{4}{(4\pi)^2} \ln\left(\frac{m_{Z'}^2}{M_G^2}\right) + \frac{1}{(4\pi)^2} \frac{10}{3} \ln\left(\frac{M_R^2}{M_G^2}\right) \tag{5}$$

where $g_c^{-2}(m_{Z'}^2)$ and $g_{1R}^{-2}(m_{Z'}^2)$ are related to the coupling constant of $Z'$ and depend on the $Z'$ mass.

## 3. COMPARISON OF VARIOUS MODELS OF THE ORIGIN OF THE $Z'$ BOSON

The $Z'$ boson field and $B^0$ gauge boson field of the $U(1)_Y$ group in the standard model are the translations of the group $O(2)$ for the $C^0$ boson field of the $U(1)_{B-L}$ group and $W_R^0$ boson field of the $U(1)_R$ group, respectively. Using the notations and formulas of Rajpoot (1980) and Langacker *et al.* (1991) we write $B^0$ and $Z'$ as

$$\begin{pmatrix} B^0 \\ Z' \end{pmatrix} = O(2) \begin{pmatrix} W_R^0 \\ C^0 \end{pmatrix} \tag{6}$$

$$O(2) = \begin{pmatrix} \dfrac{(\frac{3}{2})^{1/2} g_c}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} & \dfrac{g_{1R}}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} \\ \dfrac{g_{1R}}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} & \dfrac{-(\frac{3}{2})^{1/2} g_c}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} \end{pmatrix} \tag{7}$$

The interaction between $Z'$ and fermions is

$$\mathcal{L} = g\bar{f}[\gamma_\mu(V_f + A_f\gamma_5)]f \cdot Z'_\mu \tag{8}$$

Using the formulas of Rajpoot (1980), we obtain

$$\left.\begin{aligned} V_u &= \frac{i}{2}(g_c^2 - g_{1R}^2) \\ A_u &= \frac{i}{2} g_{1R}^2 \end{aligned}\right\} \quad \text{for up quark} \tag{9}$$

$$\left.\begin{aligned} V_d &= \frac{i}{2}(g_c^2 + g_{1R}^2) \\ A_d &= \frac{-i}{2} g_{1R}^2 \end{aligned}\right\} \quad \text{for down quark} \tag{10}$$

$$\left.\begin{aligned} V_l &= \frac{i}{2}(-3g_c^2 + g_{1R}^2) \\ A_l &= \frac{-i}{2} g_{1R}^2 \end{aligned}\right\} \quad \text{for leptons} \tag{11}$$

$$g = \frac{1}{2(g_{1R}^2 + \frac{3}{2}g_c^2)^{1/2}} \tag{12}$$

Evidently, the running coupling constants of the $Z'$ boson to fermions depend on the mass of $Z'$ only. For this reason, the width of $Z'$ decay into fermions also depends on its mass in the $SO(10)$ model:

$$\Gamma_{Z'\to f\bar{f}} = \frac{m_{Z'}}{12\pi}\left(1 - \frac{4m_f^2}{m_{Z'}^2}\right)^{1/2}\left\{[|gV_f|^2 + |gA_f|^2] + \frac{2m_f^2}{m_{Z'}^2}[|gV_f|^2 - 2|gA_f|^2]\right\} \tag{13}$$

If $m_{Z'}^2 >> 2m_f^2$, equation (13) is further reduced to

$$\Gamma_{Z'\to f\bar{f}} = \frac{m_{Z'}}{12\pi}[|gV_f|^2 + |gA_f|^2] \tag{14}$$

Using the above equations, we obtain the values and the curves of the width of the $Z'$ as shown in Table I and Fig. 2, which may be checked by further experiment. Since there are many models containing the $Z'$ boson, it is important to compare how these models interpret its origin.

To do this, we will use the strategy developed by Boudjema *et al.* (1990) (BLRV). The BLRV strategy is expressed by the curves (or strips) of $R_{5,6}$ versus $\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$ plane, where $\Gamma_{Z'\to\mu\bar{\mu}}$ is the partial width of the $Z'$ decay into the muonic pair, and

$$\Gamma_{Z'\to\Sigma_{i=1}^{5,6} q_i\bar{q}_i} \tag{15}$$

is the partial width of the $Z'$ decay into five or six known quark pairs, and

$$R_{5,6} \equiv \Gamma_{Z'\to\Sigma_{i=1}^{5,6} q_i\bar{q}_i}/\Gamma_{Z'\to\mu\bar{\mu}} \tag{16}$$

This strategy requires the preliminary measurement of the muonic pair width $\Gamma_{Z'\to\mu\bar{\mu}}$ and of the ratio $R_{5,6}$ of the $Z'$ resonance. Boudjema *et al.* (1990) only adopted the Born approximation and neglected one-loop radiative corrections, which are smaller than the experimental errors of various widths and ratios. In the $(R_{5,6}, \Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'})$ plane the two models including $Z'$ and four other models belong to completely different regions except for their $Z'_V$ boson. In order to distinguish the other three models, they further discussed the longitudinal polarized asymmetries. The direct production of a $Z'$ boson will not be seen in future $p\bar{p}$ colliders and LEP. If the $Z'$ mass is in the range

Table I.

| $m_{Z'}$ (GeV) | 500 | 600 | 700 | 800 | 900 | 1000 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 | 8000 | 9000 | 10000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Gamma_{Z'\to\mu\bar{\mu}}$ | 0.22 | 0.26 | 0.31 | 0.35 | 0.39 | 0.44 | 0.88 | 1.33 | 1.78 | 2.23 | 2.69 | 3.14 | 3.59 | 4.05 | 4.50 |
| $\Gamma_{Z'\to d\bar{d},s\bar{s},b\bar{b}}$ | 0.41 | 0.49 | 0.57 | 0.65 | 0.74 | 0.82 | 1.65 | 2.49 | 3.34 | 4.18 | 5.03 | 5.88 | 6.74 | 7.59 | 8.44 |
| $\Gamma_{Z'\to u\bar{u},c\bar{c}}$ | 0.21 | 0.14 | 0.16 | 0.19 | 0.21 | 0.23 | 0.47 | 0.71 | 0.96 | 1.20 | 1.44 | 1.69 | 1.94 | 2.18 | 2.43 |
| $R_5$ | 6.645 | 6.651 | 6.661 | 6.663 | 6.660 | 6.658 | 6.675 | 6.683 | 6.687 | 6.692 | 6.695 | 6.797 | 6.699 | 6.701 | 6.703 |
| $R_6$ | 7.173 | 7.182 | 7.190 | 7.194 | 7.192 | 7.189 | 7.209 | 7.218 | 7.223 | 7.229 | 7.232 | 7.235 | 7.237 | 7.240 | 7.242 |
| $10^3\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$ | 0.436 | 0.437 | 0.437 | 0.438 | 0.438 | 0.439 | 0.442 | 0.444 | 0.446 | 0.447 | 0.448 | 0.449 | 0.449 | 0.450 | 0.450 |
| $A_d$ | −0.657 | −0.657 | −0.657 | −0.658 | −0.658 | −0.658 | −0.658 | −0.658 | −0.658 | 0.659 | −0.659 | −0.659 | −0.659 | −0.659 | −0.659 |
| $A_u$ | −0.424 | −0.424 | −0.425 | −0.425 | −0.425 | −0.426 | −0.428 | −0.429 | −0.430 | −0.431 | 0.432 | −0.432 | −0.432 | −0.433 | −0.433 |
| $A_e$ | 0.919 | 0.919 | 0.919 | 0.918 | 0.918 | 0.917 | 0.915 | 0.913 | 0.912 | 0.911 | 0.910 | 0.909 | 0.909 | 0.908 | 0.908 |

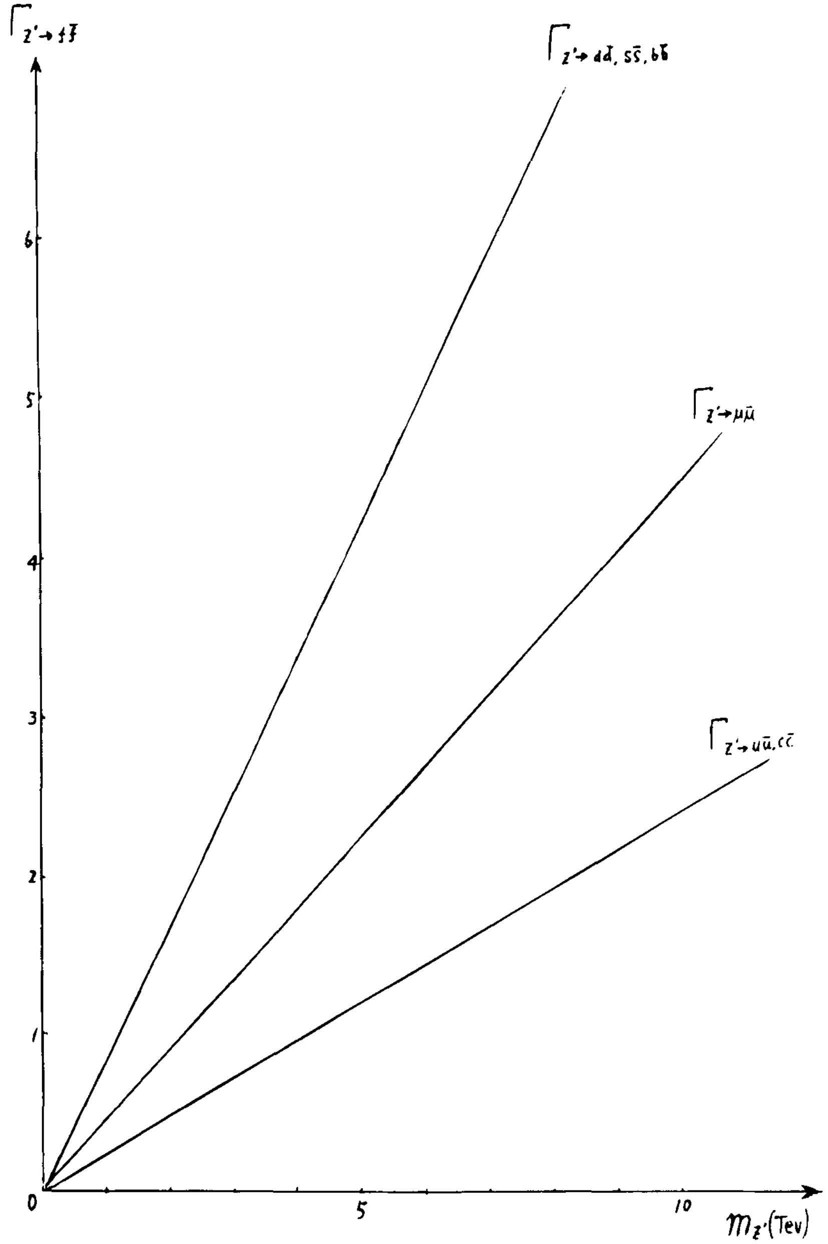


**Fig. 2.** The $\Gamma_{Z'\to f\bar{f}}$ versus $m_{Z'}$ for the $SO(10)$ model.

495 GeV $\leq m_{Z'} \leq$ 1 TeV, it might be discovered in the future $e^+e^-$ collider with total energy up to 1 TeV, and the measurement of its partial width including the top quark mass will also be possible. The BLRV strategy has been applied to six models containing $Z'$; it proved to be a good method in distinguishing these models.

In this paper we apply the BLRV strategy to the $SO(10)$ model, which has not been done yet, to our knowledge. Since $R_{5,6}$ and $\Gamma_{Z'\to\mu\bar{\mu}}$ in the $SO(10)$ model depend on the mass of $Z'$ only, it is easy to obtain the curves of the $R_{5,6}$ versus $\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$ plane by using the above equations, which is displayed in Table I and Figs. 3 and 4 when the $Z'$ mass is between 500 GeV and 10

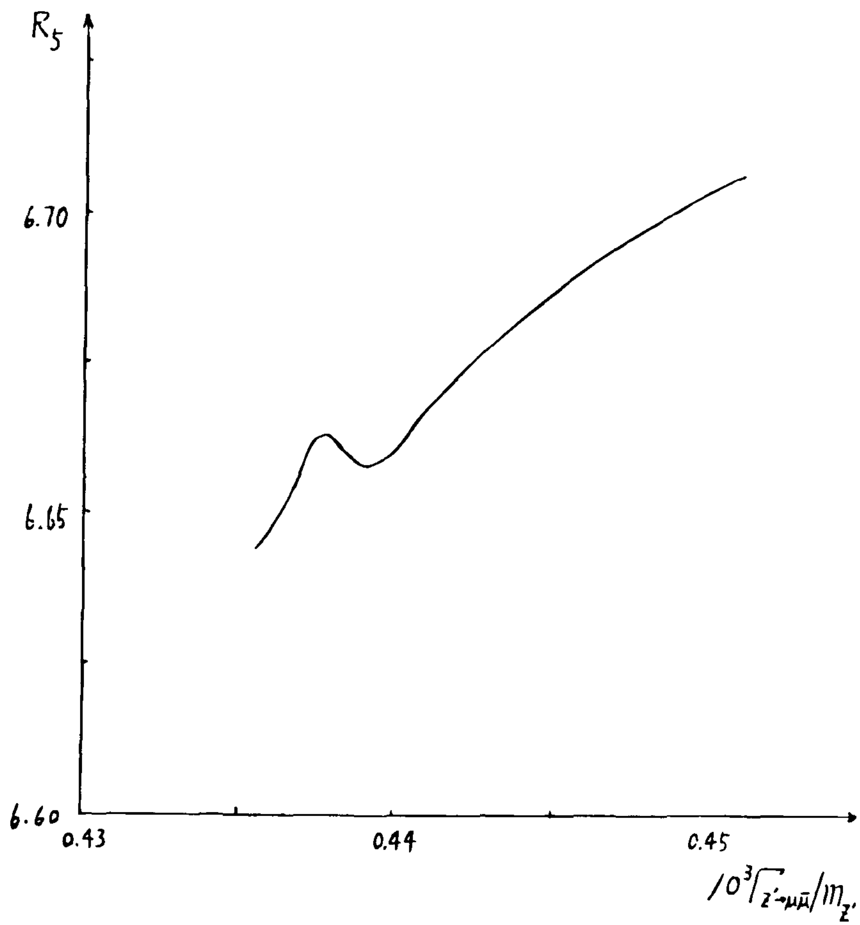


**Fig. 3.** The ratio $R_5$ versus $\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$ for the $SO(10)$ model.

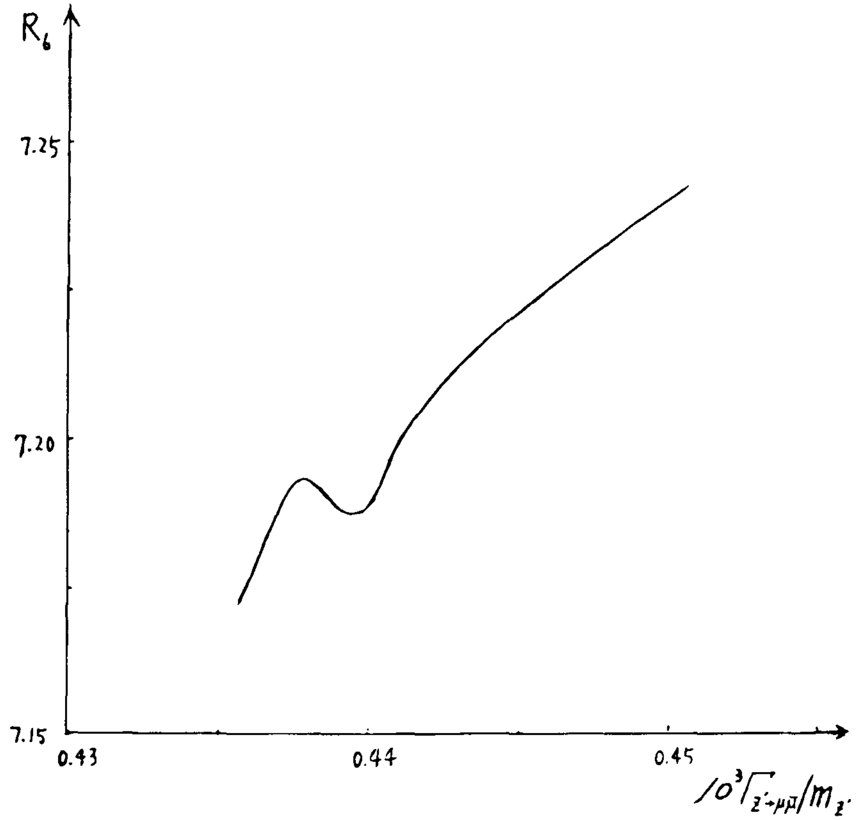


**Fig. 4.** The ratio $R_6$ versus $\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$ for the $SO(10)$ model.

TeV. In Figs. 3 and 4 the range of the variance of the curve is very small regardless of the $R_{5,6}$ or $\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$; the range of variance of the values is not more than 0.1. These values may be considered as determinate values, and can be directly compared with experimental values if the experimental errors are not over 0.1. Figures 5 and 6 compare Figs. 3(4) and Figs. 4(5) of BLRV. It is evident that the curves in Figs. 3(4), the strip of the left–right symmetric model (LRM) (Verzegnassi, 1988), and the superstring-inspired $E_6$ model (Barbieri and Hall, n.d.; Font *et al.*, 1989) have no common intersection. These results are consistent with experimental results that do not support LRM and $E_6$ models, but do support the $SO(10)$ model.

Figures 7 and 8 compare Figs. 3(4) and 6(7) of BLRV; they include four other models having the $Z'$ boson aside from RLM and $E_6$. In Figs. 7 and 8 the curves of the $SO(10)$ model and other four models having the $Z'$ boson also have no common intersection. Because the composite models ($Y$, $Y_L$, $Z^*$) have not been distinguished in Figs. 7 and 8, and are therefore

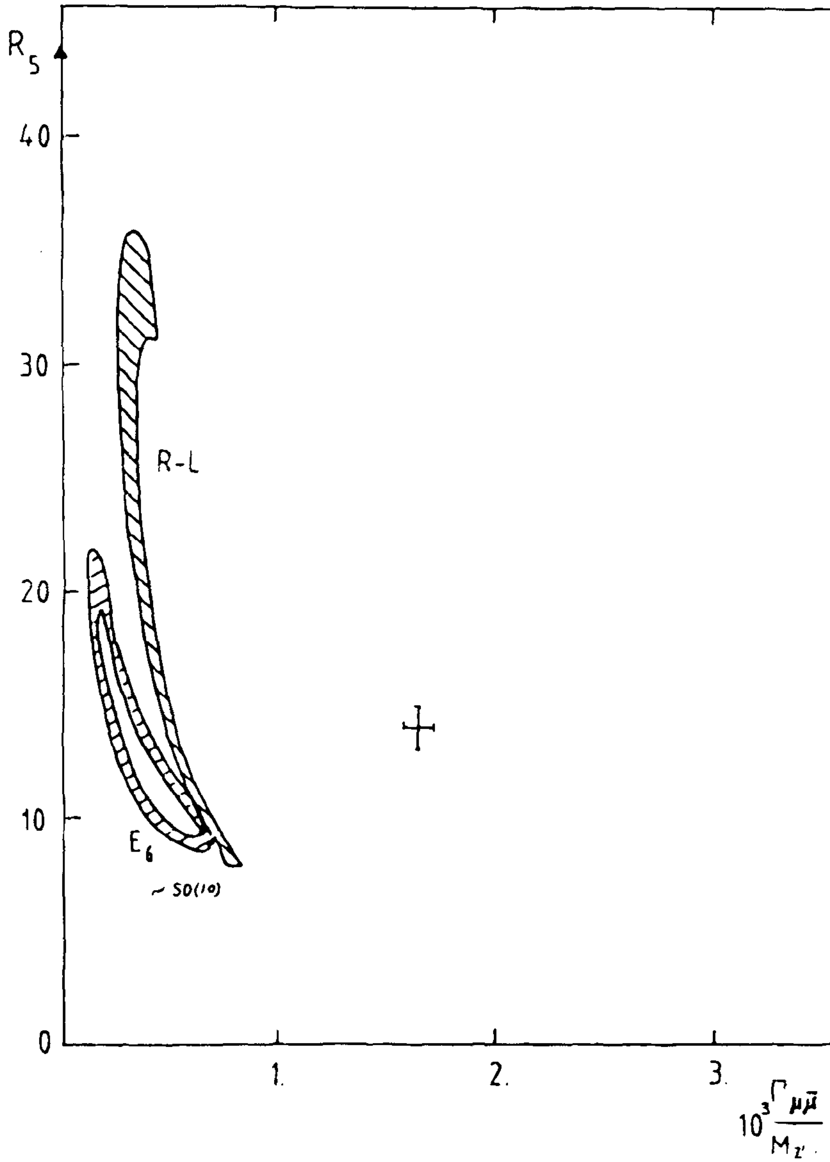


**Fig. 5.** The ratio $R_5$ versus $\Gamma_{\mu\bar{\mu}}/m_{Z'}$ for the $SO(10)$, $E_6$ and R-L models.

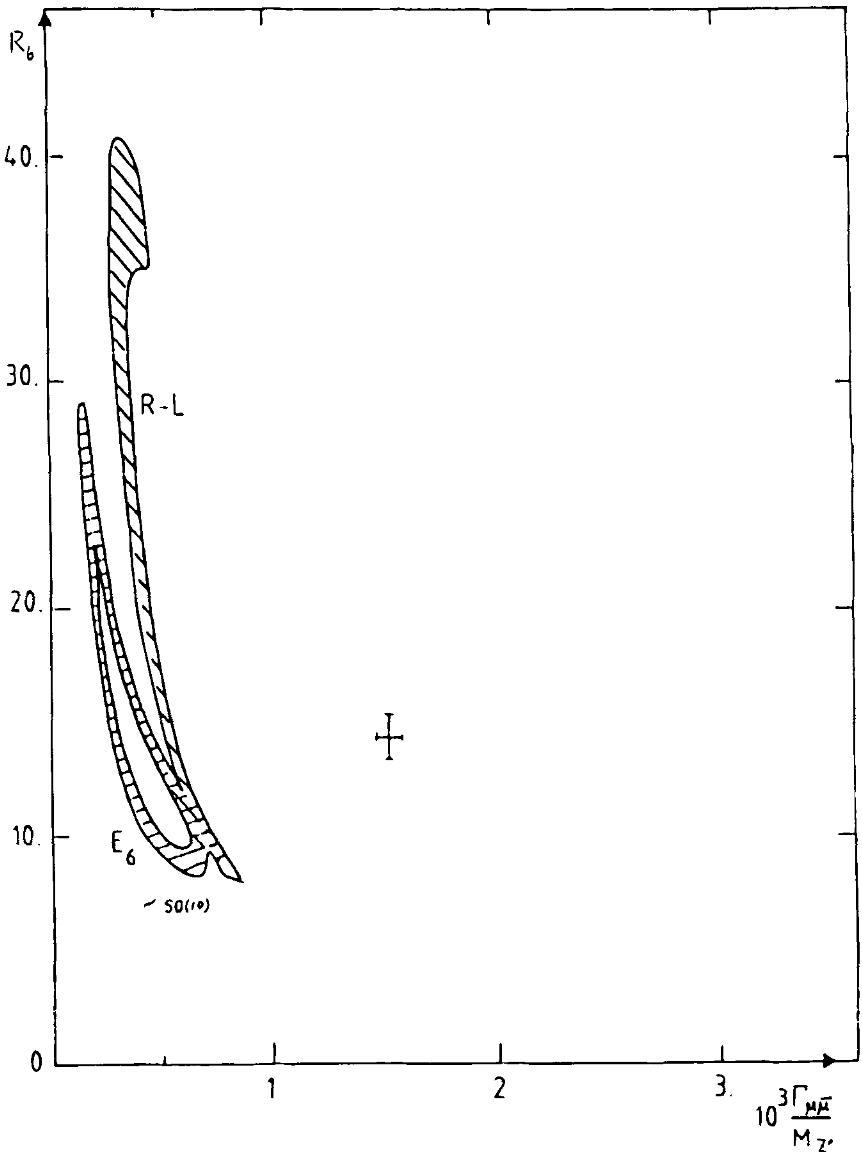


**Fig. 6.** The ratio $R_6$ versus $\Gamma_{\mu\bar{\mu}}/m_{Z'}$ for the $SO(10)$, $E_6$ and R-L models.

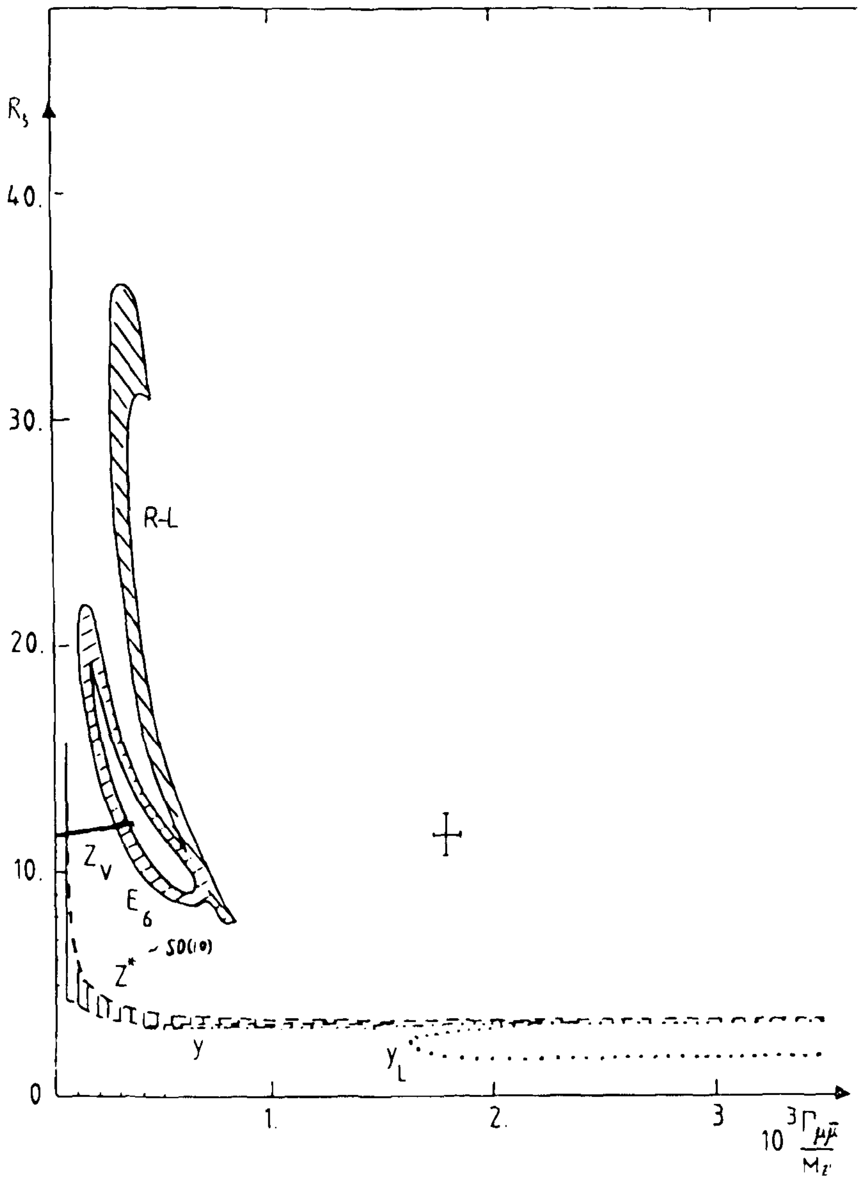


**Fig. 7.** The ratio $R_5$ versus $\Gamma_{\mu\bar{\mu}}/m_{Z'}$ for $SO(10)$ and the six considered models.

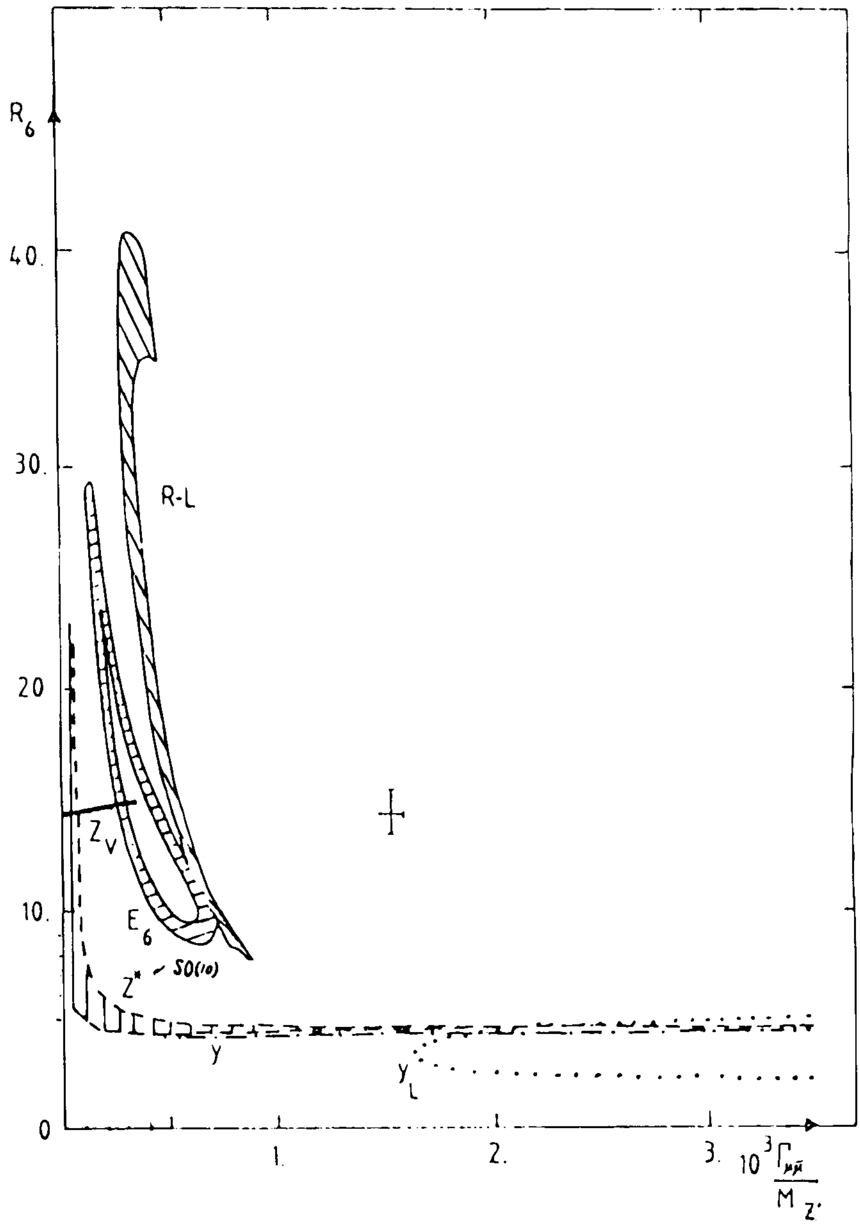


**Fig. 8.** The ratio $R_6$ versus $\Gamma_{\mu\bar{\mu}}/m_{Z'}$ for $SO(10)$ and the six considered models.

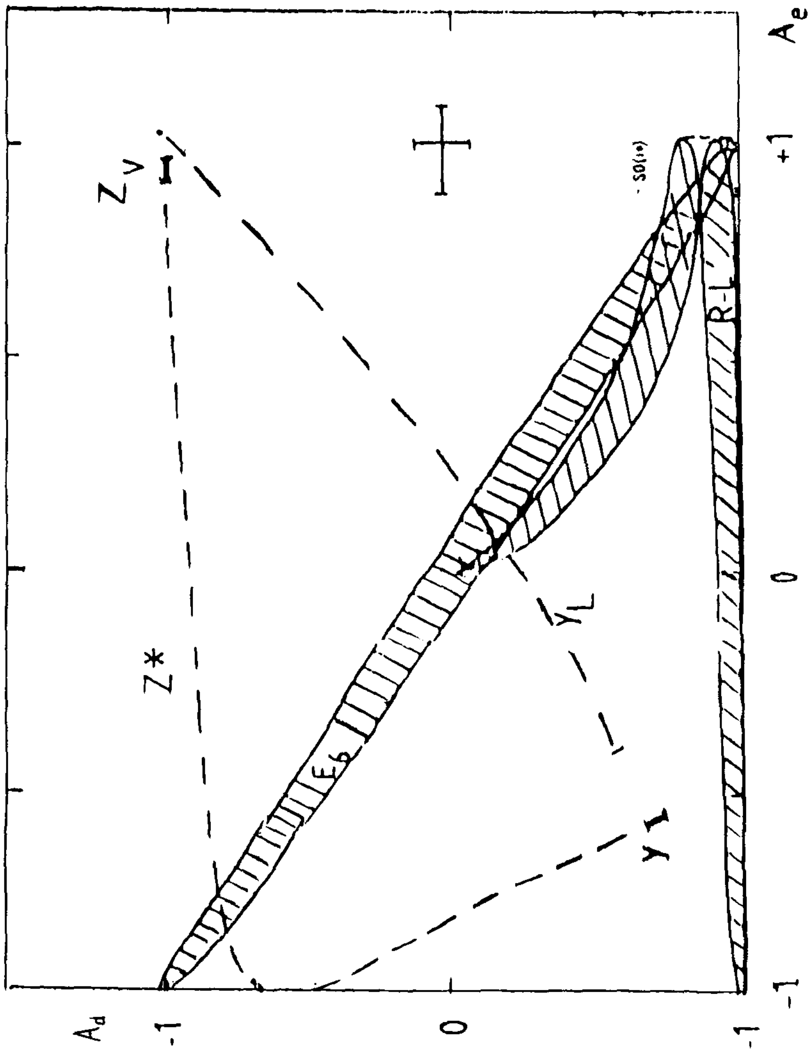


**Fig. 9.** $A_d$ versus $A_e$ for $SO(10)$ and the six considered models.

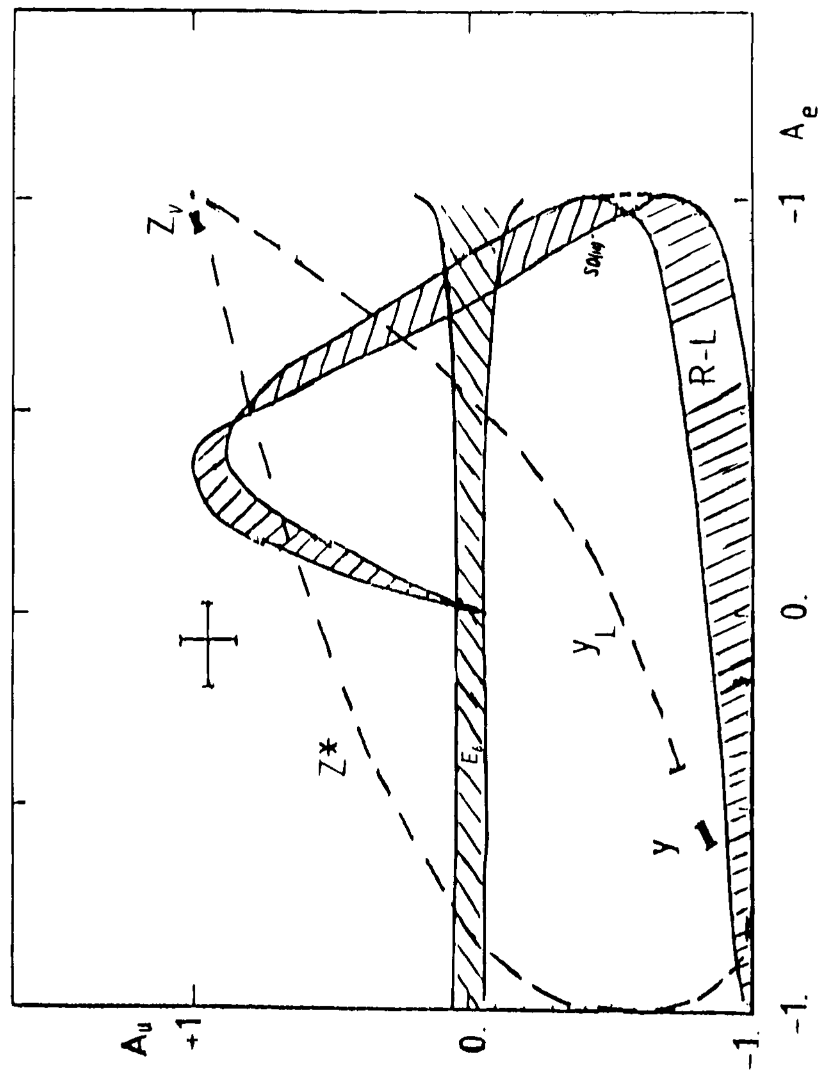


**Fig. 10.** $A_u$ versus $A_e$ for $SO(10)$ and the six considered models.

confused, BLRV further used a polarized asymmetric method to eliminate the confusion.

We will use the same method for the $SO(10)$ model to eliminate the confusion among these models. Using the method of Boudjema *et al.* (1990) and equations (8)–(11), we can obtain formulas for the three polarized asymmetries for this $SO(10)$ model. The formulas for the asymmetry in $Z' \rightarrow f\bar{f}$ follow from the tree level [details are given in Langacker *et al.* (1991) and Anderson *et al.* (1992a,b)]

$$A_{LR}^{h,so(10)} \equiv A_e^{so(10)} = \frac{N_L - N_R}{N_L + N_R} \approx \frac{V_t A_t}{A_t^2 + A_t^2} \tag{17}$$

$$A_{FB}^{u,so(10)} \equiv A_u^{so(10)} = \frac{3V_u A_u}{2V_u^2 + 2A_u^2} \tag{18}$$

$$A_{FB}^{d,so(10)} \equiv A_d^{so(10)} = \frac{3V_d A_d}{2V_d^2 + 2A_d^2} \tag{19}$$

It can be seen from equations (8)–(11) and (4), (5) that $A_e^{so(10)}$, $A_u^{so(10)}$, and $A_d^{so(10)}$ depend on the mass of $Z'$ only, and are easily calculated.

A comparison of the $A_{u,d}$ versus $A_e$ plane between the $SO(10)$ model and six other models of BLRV is given in Figs. 9 and 10, which show that the small line of $A_{u,d}^{so(10)}$ versus $A_e^{so(10)}$ for the other six models has no common intersection. These results are consistent with Figs. 5 and 8, and also indicate that the $SO(10)$ model is different from the other six models.

## 4. SUMMARY

We now summarize the $SO(10)$ model: (i) The mass scales of SSB are $M_G \approx 10^{17}$ GeV and $M_R \approx 10^9$ GeV. (ii) The mass scale of the 30 gauge bosons is about $10^{17}$ GeV. (iii) The mass scale of the $w_R^{\mp}$ bosons is about $10^9$ GeV. (iv) The range of the $Z'$ mass is 495 GeV $< m_{Z'} < 10^9$ GeV. (v) The running gauge coupling constants can be completely determined, except for $g_c(q^2)$ for $U(1)_{B-L}$ and $g_{1R}(q^2)$ for $U(1)_R$. The $g_c(m_{Z'}^2)$ and $g_{1R}(m_{Z'}^2)$ depend on the mass of $Z'$ only, and the width of $Z'$ decay into fermions also depends on the mass of $Z'$ only.

In sum, we have applied the strategy of BLRV to study the origin of $Z'$ boson in the $SO(10)$ model and six other models. The $SO(10)$ model has been supported by experiment, and is the best candidate for the unification of the electromagnetic, weak, and strong forces.

## ACKNOWLEDGMENTS

The work of T.-Z.L. was partially supported by Grant LWTZ-1298 from the Chinese Academy of Science, and the National Natural Science Foundation of china under contract number 19475049.

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